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Abstract
Modified Fermi–Dirac functions for the fermionic partons and a Bose–Einstein expression for gluons allow us to successfully describe both polarized and unpolarized structure functions in terms of a small number of parameters. Definite predictions are made for $\bar{q}$ distributions to be tested in forthcoming experiments.

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It is a great honour for me to contribute to the volume dedicated to my colleague and friend, Lochlainn O'Raifeartaigh.

The approximate scaling properties of the structure functions for deep inelastic scattering processes initiated by charged leptons or neutrinos give twofold support to QCD for its asymptotic freedom [1], which accounts for the success of the parton model, and for the soft narrowing of x-distributions, which is expected from quantum corrections [2].

The theory is able to predict the structure functions for every $Q^2 > Q_0^2$, once they are given at Q_0^2 together with the gluon distributions [2].

There are restrictions on the parton distributions, which follow from the quantum numbers of the target (proton or neutron) [3], or from the chiral properties of QCD [4]:

$$\int_0^1 [u(x) - \bar{u}(x)]\,\mathrm{d}x = 2 \tag{1}$$

$$\int_0^1 [d(x) - \bar{d}(x)]\,\mathrm{d}x = 1 \tag{2}$$

$$\int_0^1 [u^\uparrow - u^\downarrow - d^\uparrow + d^\downarrow + \cdots](x)\,\mathrm{d}x = \frac{G_A}{G_V} \tag{3}$$

where $u(x)$ and $d(x)$ are the parton distributions in the proton (isospin symmetry implies that in order to obtain those appropriate for the neutron one has to exchange $u \longleftrightarrow d$ and $\bar{u} \longleftrightarrow \bar{d}$) and the Q^2 dependence is omitted for the sake of brevity.

From equations (1)–(3) one derives the sum rules:

$$\int_0^1 F_3(x, Q^2)\,\mathrm{d}x = 3\left[1 - \frac{\alpha_S(Q^2)}{\pi} - \cdots\right] \tag{4}$$

$$\int_0^1 [g_1^p(x) - g_1^n(x)]\mathrm{d}x = \frac{1}{6}\frac{G_A}{G_V}\left[1 - \frac{\alpha_S(Q^2)}{\pi} - \cdots\right]. \tag{5}$$

The role of the Pauli principle for parton distributions was advocated many years ago by Niegawa and Sasaki [5] and by Feynman and Field [6]; these authors state

'... the pairs $u\bar{u}$ expected to occur in the small x region (the '*sea*') are suppressed more than $d\bar{d}$ by the exclusion principle... '.

They also assumed a different high-x behaviour for u and d partons to comply with the dramatic fall at large x of the ratio [7]

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u(x) + 4d(x) + \cdots}{4u(x) + d(x) + \cdots}. \tag{6}$$

This behaviour may also be a consequence of the Pauli principle, which may demand broader x-distributions for higher first moments; moreover, the increasing of the ratio at high x [8]

$$\frac{g_1^p(x)}{F_1^p(x)} = \frac{[4u^\uparrow - 4u^\downarrow + d^\uparrow - d^\downarrow](x) + \cdots}{4u(x) + d(x) + \cdots} \tag{7}$$

requires the dominance at high x of $u^\uparrow$, the valence parton with the highest first moment. In fact, if we neglect the $\bar{q}$ contributions, equations (1)–(3) and the second Bjorken sum rule imply

$$u^\uparrow \simeq \tfrac{3}{2} \simeq d^\downarrow + u^\downarrow + d^\uparrow. \tag{8}$$

By assuming $d(x) = 2u^\downarrow(x)$, which is an approximately good relationship for their first moments, one may deduce, in the region where valence quarks dominate, by neglecting the contribution of $\Delta d(x)$ to $g_1^p(x)$ [9],

$$xg_1^p(x) = \tfrac{2}{3}[F_2^p(x) - F_2^n(x)] \tag{9}$$

which is well satisfied for $x > 0.2$.

The role of the Pauli principle has motivated the description of the quark parton distributions in terms of Fermi–Dirac functions [10, 11]:

$$xp(x) = \frac{f(x)}{\exp\{[x - \tilde{x}(p)]/\bar{x}\} + 1} \tag{10}$$

where $f(x)$, $\bar{x}$ and $\tilde{x}(p)$ play the role of the *weight function*, *temperature* and *thermodynamical potential* of the quark parton p, characterized by its flavour and helicity.

The chiral QCD properties relate the potential for antiparticles with opposite helicity [11, 12]:

$$\tilde{x}_q^h + \tilde{x}_{\bar{q}}^{-h} = 0. \tag{11}$$

To describe data we assume for the light quarks (u, d) and their antiparticles [13]

$$f(x, p) = A\tilde{x}_q^h x^b \tag{12}$$

$$f(x, \bar{p}) = \frac{\bar{A}x^{2b}}{\tilde{x}_q^{-h}} \tag{13}$$

respectively. To agree with the experimental data we need to modify the Fermi–Dirac expressions for the quark by the factor $\tilde{x}_q$; it is reasonable, in this framework, to divide the expressions for the antiquarks with opposite helicity by the same factor in such a way that the product of the two expressions is the same as one should obtain with Fermi–Dirac functions. Also the factor of two for the exponents for the small-x behaviour has been assumed on an empirical basis (with this value, we shall find, $b \simeq 0.4$, which corresponds to a factor $\simeq\frac{1}{3}$ for the negative exponents for the small-x behaviour of $\bar{q}(x)$ and $q(x)$, respectively).

To comply with the small-x behaviour one has to add a diffractive term characterized at small x—by a power-like behaviour with a more negative exponent than the Fermi–Dirac term (10) [13]:

$$\frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \tag{14}$$

to be added to the modified Fermi–Dirac term defined by equations (10)–(13).

The form chosen for the diffractive term is different from the previous proposals and is strictly related to the form assumed for the gluons, with the same power-like behaviour at small x and a vanishing *thermodynamical potential*. These assumptions are consistent with the relationship, suggested by QCD, between the sea and the gluons and with the requirement that the diffractive part is unpolarized, obeys equation (11) and is invariant under charge conjugation.

In conclusion we have

$$xu^{\uparrow}(x) = \frac{A\tilde{x}_u^+ x^b}{\exp[(x - \tilde{x}_u^+)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \tag{15}$$

$$x\bar{u}^{\downarrow}(x) = \frac{\bar{A}x^{2b}}{\tilde{x}_u^+\{\exp[(x + \tilde{x}_u^+)/\bar{x}] + 1\}} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \tag{16}$$

and similar expressions for the other light partons.

Since we have no such constraints on strange partons, in order not to introduce new parameters we take from experiment [14]

$$s(x) = \bar{s}(x) = \frac{\bar{u}(x) + \bar{d}(x)}{4} \tag{17}$$

and to obey the second Bjorken sum rule we assume

$$\Delta s(x) = \Delta\bar{s}(x) = \frac{\Delta\bar{d}(x) - \Delta\bar{u}(x)}{3}. \tag{18}$$

At the scale we consider, $Q_0^2 = 4\ (\mathrm{GeV})^2$, we neglect charmed partons, which will be induced at higher Q^2 by the evolution equations.

Finally we take the Bose–Einstein form for gluons:

$$xG(x) = \frac{A_G x^{\tilde{b}+1}}{\exp\,(x/\bar{x}) - 1}. \tag{19}$$

The choice to take the highest possible value for $\tilde{x}_G^{\pm 1}$, zero, which implies $\Delta G(x) = 0$, corresponds to considering the hadrons in the deep inelastic regime as a black-body cavity for the colour gauge field.

The quantities A and $\bar{A}$ are constrained by equations (1) and (2), while A_G has to verify the momentum sum rule, so we have eight free parameters, $\bar{x}$, $\tilde{x}_u^h$, $\tilde{x}_d^h$, b, $\tilde{A}$ and $\tilde{b}$ for the quarks and none for the gluons.

By taking data from NMC, BCDMS, E665, Zeus and CCFR collaborations for the unpolarized structure functions and from SMC, E154 and E155 for the polarized ones[1], we

[1] References quoted in [13].

obtain [13] a total $\chi^2 = 322$ for the selected 233 data points with the values of the eight free parameters given by

$$\bar{x} = 0.099\,07 \tag{20}$$

$$\tilde{x}_u^+ = 0.461\,28 \tag{21}$$

$$\tilde{x}_d^- = 0.301\,74 \tag{22}$$

$$\tilde{x}_u^- = 0.297\,66 \tag{23}$$

$$\tilde{x}_d^+ = 0.227\,75 \tag{24}$$

$$b = 0.409\,62 \tag{25}$$

$$\tilde{b} = -0.253\,47 \tag{26}$$

$$\tilde{A} = 0.083\,18. \tag{27}$$

The inequalities:

$$\tilde{x}_u^+ \gg \tilde{x}_d^- \simeq \tilde{x}_u^- > \tilde{x}_d^+ \tag{28}$$

confirm the previous determinations [10, 15].

There are appealing approximate relationships for the potentials:

$$\tilde{x}_d^- \simeq \tilde{x}_u^- \simeq \tfrac{2}{3}\tilde{x}_u^+ \simeq \tfrac{4}{3}\tilde{x}_d^+. \tag{29}$$

We found higher values for $\Delta s + \Delta\bar{s}$ and still more for the non-diffractive part of $s + \bar{s}$ than the upper limits previously found [16] in the study of charm production. However, since in that case different values have been assumed for the exponents of the powers for the small-x behaviour of the two terms, the analysis should be redone with those assumed here. The inequality $\bar{d}(x) > \bar{u}(x)$, which we expect within the statistical approach, is in disagreement at high x, where the experimental errors are large, with the trend of data found by E866/Nusea collaboration [17].

Waiting for more precise experiments in this field, an important test for high-x $\bar{q}$ distributions will be supplied by the study at RHIC of the $W^\pm$ production in p–p collisions [18].

Should the statistical approach survive the crucial test just mentioned, in order to retain the quantum statistical properties displayed by parton distributions at $Q^2 > Q_0^2$ and $\Delta G = 0$, we should modify the evolution equations, as done several years ago [19], by taking into account Pauli blocking for fermions and stimulated emission for gluons. This last effect, which enhances gluon formation, may account for the highest possible value, zero, assumed for the *thermodynamical potential* of gluons of both helicities.

Conclusions

Inspired by some experimental indications in favour of a role of the Pauli principle for the fermionic partons, we have been able to construct a successful set of parton distributions in terms of a small number of parameters.

A crucial test will be performed by the determination of light $\bar{q}$ distributions at high x in $W^\pm$ production in p–p reactions at RHIC and with more precise measurements of Drell–Yan processes.

Should the predictions on $\bar{u}$ and $\bar{d}$ partons from the statistical approach

$$\bar{d}(x) - \bar{u}(x) > 0 \tag{30}$$

$$\Delta\bar{u}(x) > 0 \tag{31}$$

$$\Delta\bar{d}(x) < 0 \tag{32}$$

be confirmed by experiment, it would be worthwhile to understand the appearance of the factors $\tilde{x}_q^h$ in the numerators (denominators) of the non-diffractive terms of the light-quark distributions.

Also one should write statistically inspired strange parton distributions and modify the evolution equations in such a way as to retain for $Q^2 > Q_0^2 = (4\ \mathrm{GeV})^2$, the form proposed here for parton distributions.

A phenomenological success of quantum-statistics-inspired expressions for partons would imply that the hadrons, even in the deep inelastic regime, have a limited phase space for the contained partons, which is a necessary condition for the deviation from the extreme-dilution case, which gives rise to classical statistics. The increase with Q^2 of the transverse degrees of freedom should imply the approach to the classical limit, but the transverse momentum of a parton with a finite longitudinal momentum contributes to its energy. As a consequence the role of the transverse degrees of freedom might be less relevant than generally believed: indeed, a cut on the transverse momentum of the produced hadrons is present in high-energy hadronic reactions.

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